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LETTER TO THE EDITOR

Non-linear conductivity of granular superconductors: a novel breakdown problem

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Abstract. We study a percolating network of highly hysteretic Josephson junctions in which each junction is modelled as a piecewise Ohmic hysteretic device. At the percolation threshold, the average voltage drop across a network on a large $L \times L$ grid is assumed to fall to zero as $V \sim (I - I_c(L))^\eta$ as the applied current is reduced to the critical current $I_c(L)$. Our finite-size Monte Carlo study gives the value $\eta = 2.0 \pm 0.1$. This value is remarkably close to the result $\eta = 2$ obtained by a scaling analysis of a continuum version of the model.

There is currently much interest in the properties of random media which are changed irreversibly by an applied force [1-5]. Examples include electric breakdown in a random network of resistors which short out above a critical voltage [1], 'burn out' in a random network of fuses [2], and fracture of brittle materials represented by networks of Hookean springs with a load limit [3].

In this letter we study a new problem in this general class of 'breakdown' problems: the onset of superconductivity in a percolating network of highly hysteretic Josephson junctions which occurs as the applied current is reduced. In contrast to earlier work on granular superconductors, we are able to probe the finite-voltage response of highly disordered networks‖. We assume that the average voltage drop across a large $L \times L$ network at the percolation threshold falls to zero as $V \sim (I - I_c(L))^\eta$ as the current I is lowered to the critical current $I_c(L)$. The finite-size Monte Carlo study described below yields the value $\eta = 2.0 \pm 0.1$ for the critical exponent η in two dimensions (2D). Neither η nor analogous exponents in other breakdown problems have previously been determined. A continuum 'Swiss cheese' version of the model is then studied using an approximate scaling approach and is found to have $\eta = 2$ in all dimensions $d \geqslant 2$. Finally, we show how this theory can be applied to a variety of continuum breakdown problems.

In our discrete model, each site of a square lattice is occupied by a superconducting 'grain' with probability p and by an insulating grain with probability $1 - p$. Adjacent grains are connected by Josephson junctions. We neglect the detailed phase dynamics

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‖ The zero-voltage limit of random superconducting networks is well understood: see, for example, [6] and references therein. Also note that the reversible non-linear conductivity of metal-insulator mixtures at the percolation threshold has recently been studied [7].

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of the problem and instead model the junctions by piecewise Ohmic hysteretic devices. To be specific, we assume that a single junction behaves as a Ohmic resistor with unit resistance as the applied current is reduced from infinity. Once the junction critical current I_c^0 is reached, the voltage drops irreversibly to zero. Thus, for large applied currents the network is in the purely Ohmic state. As the current is reduced, a junction goes superconducting at some point and the resistance of the network drops discontinuously. More and more junctions switch into the zero voltage state as the applied current is further reduced, until at the critical current for the network the voltage drops to zero. It is important to note that once a junction has become superconducting in our model, it remains in this state even if the current through it exceeds I_c^0 at some later time. Although the I-V curve for a particular network is discontinuous, it becomes continuous when averaged over an ensemble of networks. The average I-V curve obtained in this way governs the behaviour of the system only when the applied current is being reduced: the resistivity remains constant if the applied current is increased.

There are several approximations inherent in this model. Real Josephson junctions have a second critical current $I_c' > I_c^0$ above which they must return to the Ohmic state; however, if $I_c' \gg I_c^0$ we may safely ignore the possibility that some junctions could temporarily revert to the resistive state as the applied current is lowered. In addition, the resistive branch of the junction I-V curve is not Ohmic close to the critical current. Thermal and quantum fluctuations are neglected entirely. Finally, the most serious failing of our model is its crude treatment of the non-linear phase dynamics of the network of Josephson junctions. However, the full non-linear dynamics problem for a collection of junctions has only been studied for three coupled junctions [8], and for a chain of coupled junctions [9]. Our model, albeit simple, should display some of the features of real granular superconducting thin films at finite voltages.

Let us first consider the behaviour of an infinite square network for $p > p_c \simeq 0.592\,77$. Initially the applied voltage is large and the current passing through each junction is the same as if the junction I-V curves were purely Ohmic. Therefore the resistivity of the array ρ scales like $\rho \sim (p-p_c)^t$ for small positive $p-p_c$ when the voltage is large. (In 2D, $t \simeq 1.3$.) As the applied voltage is reduced the resistivity drops until it falls to zero at some critical current density $j_c(p)$. The nodes-and-links approximation [10-13] predicts that

$$j_c \sim (p-p_c)^\nu \qquad \text{as } p \to p_c. \tag{1}$$

This is in good agreement with Monte Carlo results [12, 13] and with critical current measurements on thin film superconductor-semiconductor mixtures [14].

At the percolation threshold, the critical current density j_c is zero in the infinite-size limit. For finite systems, a simple scaling argument combined with the nodes-and-links result (1) shows that $j_c(L) \sim L^{-1}$ for large L. The voltage drop per unit length across a large network falls to zero as $[j-j_c(L)]^\eta$ as the applied current density j is reduced toward $j_c(L)$. To obtain an estimate of the exponent η we will use finite-size scaling to extrapolate our Monte Carlo results on $L \times L$ arrays to the $L = \infty$ limit. It is natural to assume that estimates of η obtained for $L \times L$ arrays, denoted $\eta(L)$, may be extrapolated to $L = \infty$ using the scaling form $\eta(L) \sim \eta(1 + AL^{-1} + BL^{-\Delta} + \ldots)$, just as in finite-size studies of equilibrium critical phenomena [15].

To obtain the required finite-size estimates of η, we randomly generated junction networks on $L \times L$ square grids with $L = 6, 8, 10, 12, 16$ and 24 at $p = p_c$. Superconducting grains in the top and bottom rows of a particular array were connected by Josephson junctions to terminals at voltage V and zero respectively. Finally, the network was discarded and another was generated if no conducting pathway ran from terminal to terminal.

The next step was to construct the $I-V$ curve for this array. Dangling bonds carry no current and so were removed using the 'burning' algorithm of Herrmann *et al* [16]. The terminal voltage V was then set at a high initial value and Kirchhoff's laws were solved for the voltages on the backbone sites with all the junctions in the Ohmic state. In some networks, one or more bonds with special geometrical symmetries carried no current. The nodes at either end of these bonds were merged to form single superconducting grains, and Kirchhoff's laws were again solved. This process was repeated until all bonds carried non-zero current, and then the initial value of the network resistivity was recorded.

The linearity of Kirchhoff's laws show that as the applied voltage is reduced, the $I-V$ curve remains linear until a junction goes superconducting. Once the first junction which switches to the superconducting state and the applied voltage at which this occurred were found, the sites at either end of this bond were merged into a single superconducting node. Kirchhoff's laws were solved again at this applied voltage and any new bonds which had current less than I_c^0 passing through them were switched to the zero voltage state. The applied voltage was then lowered until another junction went superconducting, and the voltage was again held constant until junctions stopped switching. This processes was repeated until the entire network had switched into the superconducting state.

The $I-V$ curves obtained in this way vary greatly from sample to sample, so curves for a very large ensemble of 2^{14} networks were averaged together. The resulting quasicontinuous $I-V$ curves yield values for the critical current densities $j_c(L)$ which compare favourably with the nodes-and-links prediction $j_c(L) \sim L^{-1}$. A finite-size estimator $\eta(L)$ for η was obtained by fitting the averaged $I-V$ curve for $L \times L$ networks to the form $V \sim [j - j_c(L)]^{\eta(L)}$ over the interval $0.005 \leqslant j - j_c(L) \leqslant 0.1$. To obtain an error estimate for $\eta(L)$, we divided the ensemble into 32 sets of 512 networks. A value of $\eta(L)$ was obtained for each of these sets, and the standard deviation of these values was adopted as the error in $\eta(L)$. The results (figure 1) were extrapolated to $L = \infty$ using our scaling ansatz and assuming that $\Delta > 1$. This yielded the final result $\eta = 2.0 \pm 0.1$.

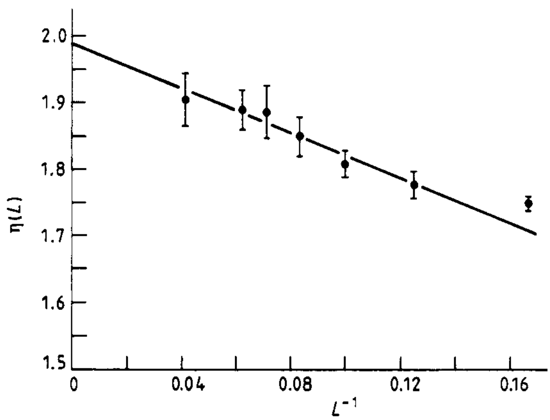

Figure 1. Values of $\eta(L)$ plotted against L^{-1}. The straight line is a linear least-squares fit to the data for $L \geqslant 8$.

To gain some insight into this result, we apply the nodes-and-links approximation [10, 11] to a continuum 'Swiss cheese' version of the discrete problem†. In the continuum problem, discs of radius a of a non-linear, hysteretic conductor are placed randomly in a perfectly insulating 2D medium. The conducting material has resistivity ρ_0 as long as the current density passing through it remains above the critical value $j_{0,c}$, but when j drops below $j_{0,c}$, the resistivity drops irreversibly to zero. Thus, the current–voltage relationship for this non-linear, hysteretic conductor is the same as that for a regular array of our 'Josephson junctions'. The continuum problem can be thought of as a disordered granular network in which the length scale of the disorder is much larger than the grain size; in the original discrete problem these two lengths are comparable.

When the applied voltage is large and the volume fraction of conductive material p exceeds the critical value p_c, all of the conducting material in the continuum model is in the Ohmic state. The nodes-and-links analysis of Halperin *et al* [18] predicts that in this regime the resistivity scales like $\rho \sim (p-p_c)^t$ with $t=1$ in $d=2$. This is the same value of t obtained when the nodes-and-links approximation is applied to the usual random resistor problem, so we might expect our continuum and discrete models to have similar behaviour close to their respective critical currents.

In the low-voltage limit, the resistance of the system will come entirely from 'bottlenecks' at the intersection of discs with small overlap. We first compute the current-dependent resistance $R(I)$ of one such constriction with minimum width $\delta \ll a$ (see figure 2). For currents I just above the critical current for the constriction $j_{0,c}\delta$, the length of the resistive region $2x_0$ is small. As a first approximation, we take the current density to be independent of y within the bottleneck. The length $2x_0$ is then given by $2j_{0,c}y(x_0)=I$, where $y(x)>0$ is the y coordinate of the disc in the first quadrant. When x_0 is small, $2y(x)\simeq \delta+4ax/\delta$ for $0\leqslant x\leqslant x_0$, so $x_0\simeq(4aj_{0,c})^{-1}\delta(I-j_{0,c}\delta)$. Using these results, we find that the resistance of the bottleneck is

$$R(I)=\int_0^{x_0}\frac{\rho_0\,\mathrm{d}x}{y(x)}\simeq\frac{\rho_0}{2a}\left(\frac{I}{j_{0,c}}-\delta\right) \qquad (2)$$

for $\delta \ll a$ and $I-j_{0,c}\delta$ small and positive.

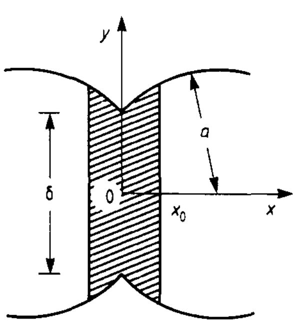

Figure 2. Intersection of two discs of radius a showing the minimum width of the constriction δ. The region in the Ohmic state (hatched area) has length $2x_0$.

† Ideally, one would like to use this approximation to study the discrete problem. However, the I–V curve for the percolating network has the same form as that for a single junction in this approximation. Presumably an improved version of the nodes-and-links picture, such as the links-and-blobs approach [17], is needed. Unfortunately, not enough is known about blob structure to carry out this calculation.

We next must determine how the bottleneck resistances are distributed and how they combine to give the resistivity of the entire network. In the nodes-and-links picture, the conducting backbone is composed of a network of quasi-one-dimensional 'strings' which join a set of nodes whose typical separation is the correlation length $\xi \sim (p-p_c)^{-\nu}$. Each string consists of a tortuous path of $L_1+1 \sim (p-p_c)^{-1}$ overlapping discs connected in series. If we ignore correlations within the backbone, the probability that the width of the constriction formed by overlapping discs is δ is $p(\delta)=\frac{1}{2}\delta/a^2$. According to the sampling theory of order statistics [19], the mean minimum value $\delta_{\min}$ of a large sample of L_1 widths δ with this distribution is $\delta_{\min} \simeq 2a^2/L_1$. The mean critical current for the string is therefore $I_{c,\text{string}} \simeq 2j_{0,c}a^2/L_1$.

We now estimate the current-dependent resistance of the string, $R_{\text{string}}(I)$, given the value of $\delta_{\min}$. For $p-p_c$ small, L_1 is large and a sum over the constrictions may be replaced by an integral:

$$R_{\text{string}}(I) \simeq L_1 \int_{\delta_{\min}}^{I/j_{0,c}} \frac{\rho_0}{2}\left(\frac{I}{j_{0,c}}-\delta\right) p(\delta)\,\mathrm{d}\delta. \tag{3}$$

The small δ cutoff in this integral ensures that the string resistance falls to zero at the string critical current†. Inserting our expression for $p(\delta)$ in (3), we obtain $R_{\text{string}}(I) \sim L_1\delta_{\min}(I-j_{0,c}\delta_{\min})^2$ for $I \to I_{c,\text{string}}$. The resistivity of the entire network is therefore $\rho(j) \sim R_{\text{string}}(\xi j) \sim (p-p_c)^{-2\nu}[j-j_c(p)]^2$ for $p-p_c$ small and $j \to j_c(p)^+$. The critical current density scales like $j_c(p) \sim (p-p_c)^{\nu+1}$ as $p \to p_c^+$. This differs from the nodes-and-links result for the discrete problem (equation (1)) because arbitrarily small constriction widths δ can occur in the continuum problem. Finally, for small but fixed $p-p_c>0$,

$$V/L = \rho(j)j \sim (p-p_c)^{-\nu+1}[j-j_c(p)]^2 \tag{4}$$

as $j \to j_c(p)^+$.

A standard scaling argument now shows that for $p=p_c$ and large but finite L, the voltage drop per unit length scales as $V \sim L^{2-1/\nu}[j-j_c(L)]^2$ as $j \to j_c(L)^+$, and that $j_c(L) \sim L^{-(1+1/\nu)}$. We thus have our final result $\eta=2$ in $d=2$, a result which is remarkably close to our Monte Carlo result $\eta=2.0\pm0.1$ for the discrete problem.

Our nodes-and-links treatment of the continuum problem is readily extended to dimensions higher than 2, and we find that $\eta=2$ for all $d>1$. It would be most interesting to investigate whether this 'superuniversality' of the exponent η also occurs in our original discrete model.

Our approach can also be applied to a rather different problem in which insulating spheres are randomly embedded in a d-dimensional homogeneous medium of our non-linear, hysteretic conducting material. As argued by Halperin *et al* [18], the conductivity exponent t for this problem differs from that in which conducting spheres are randomly embedded in an insulating medium for all $d \geq 3$. However, the exponent t is believed to be the same for these two problems in 2D. In the nodes-and-links approximation the exponents η for these problems differ even in $d=2$: arguments paralleling those given above show that the insulating disc problem has $\eta=3/2$, in contrast to our result $\eta=2$ for the conducting disc problem.

Finally, our theory is readily adapted to a variety of continuum breakdown problems. For example, consider a conducting material which has resistivity ρ_1 as long as the current density remains below a critical value $j_{0,c}$. Once the current density rises above

† The distribution $p(\delta)$ in (3) should really be replaced by the conditional probability that the width is δ, given that $\delta \geq \delta_{\min}$. Since $\delta_{\min}$ is small, this introduces only a small correction.

$j_{0,c}$, however, the material 'burns out' and the resistivity increases irreversibly to $\rho_2 \gg \rho_1$. We randomly place hyperspheres of this material in a perfectly insulating d-dimensional medium, and so obtain a continuum variant of the random fuse model studied by de Arcangelis *et al* [2]. In contrast to these authors, however, we consider the limit $\rho_1 \to 0$ rather than $\rho_2 \to \infty$. If the applied voltage is increased continuously in an infinite system for $p > p_c$, then the voltage drop per unit length is again given by (4) for j just above $j_c(p)$, and $j_c(p) \sim (p - p_c)^{(d-1)(\nu+1)}$ as $p \to p_c^+$. Similar considerations apply to a continuum version of the model for dielectric breakdown introduced by Takayasu [1]. We hope that these results will lead to Monte Carlo studies of the non-linear conductivity in other irreversible breakdown problems.

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